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TARGET ACQUISITION CONTOURS GENERATED FOR RCAG/BUFC SITES IN TH--ETC(U)

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TARGET ACQUISITION CONTOURS GENERATED FOR RCAG/BUEC SITES IN CONUS

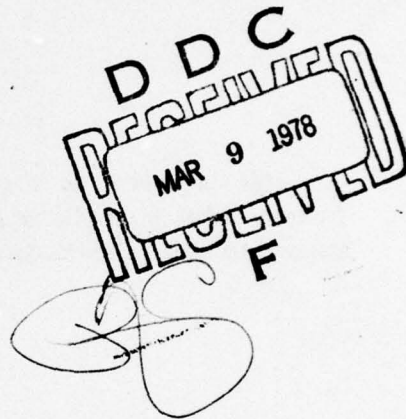
IIT Research Institute
Under Contract to
DEPARTMENT OF DEFENSE
Electromagnetic Compatibility Analysis Center
Annapolis, Maryland 21402

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November 1977

FINAL REPORT



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Prepared for

U.S. DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION
Systems Research & Development Service
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16. Abstract <p>This report documents the program, procedures and parameters used in the production of target acquisition contour overlays. These overlays, of the Remote Communications Air/Ground and Backup Emergency Communications sites in the contiguous United States, were produced for the FAA.</p>		
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PREFACE

The Electromagnetic Compatibility Analysis Center (ECAC) is a Department of Defense facility, established to provide advice and assistance on electromagnetic compatibility matters to the Secretary of Defense, the Joint Chiefs of Staff, the military departments and other DoD components. The Center, located at North Severn, Annapolis, Maryland 21402, is under executive control of the Assistant Secretary of Defense for Communication, Command, Control, and Intelligence and the Chairman, Joint Chiefs of Staff, or their designees, who jointly provide policy guidance, assign projects, and establish priorities. ECAC functions under the direction of the Secretary of the Air Force and the management and technical direction of the Center are provided by military and civil service personnel. The technical operations function is provided through an Air Force sponsored contract with the IIT Research Institute (IITRI).

This report was prepared for the Systems Research and Development Service of the Federal Aviation Administration in accordance with Interagency Agreement DOT-FA70WAI-175, as part of AF Project 649E under Contract F-19628-78-C-0006, by the staff of the IIT Research Institute at the Department of Defense Electromagnetic Compatibility Analysis Center.

To the extent possible, all abbreviations and symbols used in this report are taken from American Standard Y10.19 (1967) "Units Used in Electrical Science and Electrical Engineering" issued by the USA Standards Institute.

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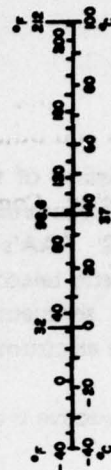
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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures			
Symbol	When You Know	Multiply by	To Find
LENGTH			
in	inches	2.5	centimeters
ft	feet	30	centimeters
yd	yards	0.9	meters
mi	miles	1.6	kilometers
AREA			
in ²	square inches	6.5	square centimeters
ft ²	square feet	0.09	square meters
yd ²	square yards	0.8	square meters
mi ²	square miles	2.6	square kilometers
	acres	0.4	hectares
MASS (weight)			
oz	ounces	28	grams
lb	pounds	0.45	kilograms
	short tons (2000 lb)	0.9	tonnes
VOLUME			
teaspoon	teaspoons	5	milliliters
fl oz	fluid ounces	15	milliliters
c	cups	30	milliliters
pt	pints	0.24	liters
qt	quarts	0.47	liters
gal	gallons	0.96	liters
ft ³	cubic feet	3.8	liters
yd ³	cubic yards	0.03	cubic meters
		0.76	cubic meters
TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature
Approximate Conversions from Metric Measures			
Symbol	When You Know	Multiply by	To Find
LENGTH			
mm	millimeters	0.04	inches
cm	centimeters	0.4	inches
m	meters	3.3	feet
km	kilometers	1.1	yards
		0.6	miles
AREA			
cm ²	square centimeters	0.16	square inches
m ²	square meters	1.2	square yards
km ²	square kilometers	0.4	square miles
ha	hectares (10,000 m ²)	2.5	acres
MASS (weight)			
g	grams	0.035	ounces
kg	kilograms	2.2	pounds
t	tonnes (1000 kg)	1.1	short tons
VOLUME			
ml	milliliters	0.03	fluid ounces
l	liters	2.1	pints
l	liters	1.06	quarts
l	liters	0.26	gallons
m ³	cubic meters	36	cubic feet
m ³	cubic meters	1.3	cubic yards
TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature



*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 236, Units of Weights and Measures, Price \$2.25, SO Catalog No. C13.10-286.

**FEDERAL AVIATION ADMINISTRATION
SYSTEMS RESEARCH AND DEVELOPMENT SERVICE
SPECTRUM MANAGEMENT STAFF**

STATEMENT OF MISSION

The mission of the Spectrum Management Staff is to assist the Department of State, Office of Telecommunications Policy, and the Federal Communications Commission in assuring the FAA's and the nation's aviation interests with sufficient protected electromagnetic telecommunications resources throughout the world to provide for the safe conduct of aeronautical flight by fostering effective and efficient use of a natural resource--the electromagnetic radio-frequency spectrum.

This objective is achieved through the following services:

- Planning and defending the acquisition and retention of sufficient radio-frequency spectrum to support the aeronautical interests of the nation, at home and abroad, and spectrum standardization for the world's aviation community.
- Providing research, analysis, engineering, and evaluation in the development of spectrum related policy, planning, standards, criteria, measurement equipment, and measurement techniques.
- Conducting electromagnetic compatibility analyses to determine intra/inter-system viability and design parameters, to assure certification of adequate spectrum to support system operational use and projected growth patterns, to defend the aeronautical services spectrum from encroachment by others, and to provide for the efficient use of the aeronautical spectrum.
- Developing automated frequency-selection computer programs/routines to provide frequency planning, frequency assignment, and spectrum analysis capabilities in the spectrum supporting the National Airspace System.
- Providing spectrum management consultation, assistance, and guidance to all aviation interests, users, and providers of equipment and services, both national and international.

EXECUTIVE SUMMARY

The Federal Aviation Administration (FAA) requested the Electromagnetic Compatibility Analysis Center (ECAC) to investigate the effects of terrain on air-ground communications between air traffic controllers and aircraft. ECAC produced terrain-dependent coverage overlays at various altitudes for each Remote Communications Air/Ground (RCAG) and Backup Emergency Communications (BUEC) site in the contiguous United States.

The ECAC Target Acquisition Model (TAM) was used to produce acquisition overlays for each RCAG/BUEC site. Data pertinent to each site was acquired from ECAC data files. This data was verified and updated by the FAA before each site was run through the TAM program. Aircraft altitudes above mean sea level and topographic data for 360 one-degree radials around the site out to 100 nautical miles were used as inputs to the TAM.

The acquisition overlays were photo-reduced over standard World Aeronautical Charts. The photo-reduced overlays were then combined into nine books, one for each FAA region in the contiguous United States.

TABLE OF CONTENTS

<u>Subsection</u>		<u>Page</u>
	SECTION 1	
	INTRODUCTION	
BACKGROUND		1
OBJECTIVE		1
APPROACH		1
	SECTION 2	
	MODEL DESCRIPTION	
TAM DESCRIPTION		2
	SECTION 3	
	ANALYSIS	4
	SECTION 4	
	RESULTS	6
	LIST OF ILLUSTRATIONS	
<u>Figure</u>		
1	SAMPLE TAM ACQUISITION CONTOURS	3
	LIST OF TABLES	
<u>Table</u>		
1	NUMBER OF RCAG AND BUEC SITES ANALYZED PER REGION	6
	LIST OF APPENDIXES	
<u>Appendix</u>		
A	PROFILE GENERATION	7
B	LINE-OF-SIGHT DETERMINATION	21

SECTION 1

INTRODUCTION

BACKGROUND

The Federal Aviation Administration (FAA) wanted to determine how air-ground communication between air traffic controllers and aircraft was affected by the terrain around the ground-based transmitter site. The Electromagnetic Compatibility Analysis Center (ECAC) was requested by the FAA to determine the terrain coverage for all Remote Communications Air/Ground (RCAG) and Backup Emergency Communications (BUEC) sites existing in the contiguous United States.

OBJECTIVE

The objective of this analysis was to produce contours that depict where radio communications with aircraft can be maintained at various altitudes around the RCAG/BUEC sites within the contiguous United States.

APPROACH

The ECAC Target Acquisition Model (TAM), in conjunction with the CALCOMP plotter, was used to produce coverage overlays. The coverage included a line-of-site contour and acquisition contours for aircraft altitudes beginning with the first even thousand foot contour above the site elevation and every 2,000 feet thereafter up to a maximum of 20,000 feet. Terrain data was obtained from ECAC's topographic files.

SECTION 2

MODEL DESCRIPTION

TAM DESCRIPTION

The Target Acquisition Model (TAM) was developed to generate acquisition contours around specified site locations. These contours are used to assess site suitability or vulnerability for the detection of aircraft.

TAM uses discrete terrain data to develop radar acquisition contour information. For a given site location, the definition of the terrain surrounding the site, and a target altitude (above sea level or above terrain), the program generates terrain elevation profiles along a series of equally spaced radials emanating from the site. The elevations are corrected for the effects of atmospheric refractivity and the earth's curvature. The ranges at which targets, approaching the site at a given altitude, would first be detected by radar is determined by analyzing these profiles. Ranges from each profile are then consolidated to produce an acquisition contour around the site for the given altitude. APPENDIXES A and B describe the analytical procedures used in TAM for profiling and line-of-sight calculations.

When scaled and reproduced on a transparent plastic sheet, the acquisition contours around the site become an actual map overlay.

When the overlay is properly oriented on an identically scaled map, the contour lines for the site indicate the distance to the points where an aircraft at the given altitude will be within line-of-sight. FIGURE 1 is a sample overlay.

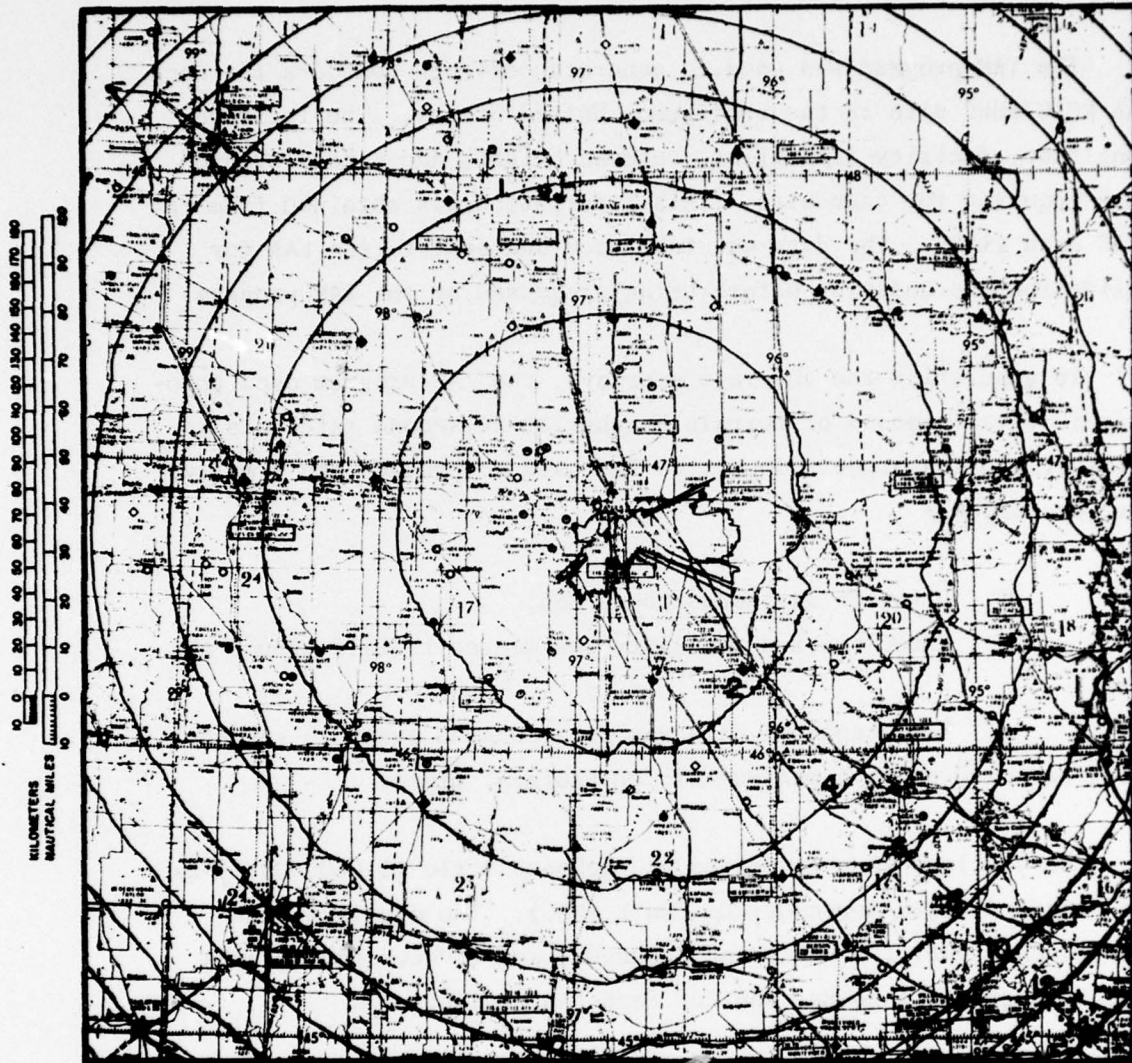


FIGURE 1. SAMPLE TAM ACQUISITION CONTOURS.

SECTION 3

ANALYSIS

The TAM program was used to generate coverage contours for each FAA RCAG/BU EC site in the contiguous United States. The latitude, longitude, facility identifier, antenna height, and site elevation were required for each site. This information was obtained from the ECAC data files. The data was tabulated and sent to the FAA for validation and updating before being processed by the TAM program.

In generating the coverage overlays, the TAM program used topographic data composed of terrain heights at 30-second intervals in latitude and longitude. Inputs to TAM used to produce the overlays were as follows:

1. Aircraft altitudes above MSL.
2. Three hundred and sixty one-degree radials around the site.
3. Topographical data was interpolated every 15 seconds along each radial to a maximum of 100 nautical miles.

The overlays were generated at the same scale (1,000,000:1) as the standard World Aeronautical Chart (WAC). The overlays were photographed over the WAC's and reduced. The reduced photographs were arranged into books, one for each FAA region.

The book for each region was organized alphabetically beginning with the Air Route Traffic Control Centers (ARTCC) in the region. The RCAG/BU EC sites serving each ARTCC were then alphabetized. RCAG/BU EC sites within a region but serving other region's ARTCC's completed the listing and were organized in the same manner.

In interpreting the coverage contours, the user should consider the following limitations. The topography of CONUS was digitized at 30-second intervals. When an elevation along a radial profile lay between digitized topographic data, a two-way linear interpolation was computed with the four adjacent data points to recover the elevation (see APPENDIX A). In rugged terrain, the digitized data and the interpolation process can overlook mountains or valleys that lay between recorded topographic elevations. Also, the topographic data file does not reflect man-made obstructions.

A single atmospheric refractivity index (310) was used for all of CONUS. The refractivity index was used to alter the earth's curvature by increasing the earth's radius. This change of radius compensates for the bending of radio waves propagated through the atmosphere. Along the profiles, elevations are also corrected using the adjusted radius (see APPENDIX B).

Frequency, power, antenna gain, etc. are not used in developing the TAM contours. The contours are based on geometric considerations. Consequently, the contours represent the maximum possible range a target could be detected at a given altitude.

SECTION 4

RESULTS

RESULTS

Nine books containing coverage contours for RCAG/BUEC sites in each of the nine FAA regions in the contiguous United States, were delivered under separate cover. TABLE 1 shows the number of RCAG sites and the number of BUEC sites analyzed for each of the nine FAA regions.

TABLE 1
NUMBER OF RCAG AND BUEC SITES ANALYZED
PER REGION

Sites Serving ARTCC's Within the Region			Sites Within the Region Serving Other Region's ARTCC's		Total
Region	RCAG	BUEC	RCAG	BUEC	
NE	18	4	1	0	23
EA	39	17	17	3	76
SO	71	2	14	1	88
GL	120	8	6	0	134
CE	29	3	19	2	53
SW	75	6	9	0	90
RM	55	7	13	1	76
NW	16	6	7	0	29
WE	30	15	10	0	55

- Notes: 1. RCAG/BUEC sites were counted as RCAG sites.
2. Duplicate sites, in a given region, were only counted once.

APPENDIX A

PROFILE GENERATION

A profile is generated between two points by calculating elevation values through interpolation of topographic data at a given increment along a great circle path from the first point to the second. A thirty second interval results in elevations being calculated approximately every half mile along the path, while a fifteen second interval yields elevations at approximately every quarter mile.

GREAT CIRCLE PATH COMPUTATION

Calculation of coordinates along a great circle path is based on a unit sphere with the coordinate system shown in Figure A-1. The i, j, and k vectors are of unit length in the X, Y, and Z directions, respectively.

Transformations involved are:

a. Spherical to rectangular:

$$X = \cos \phi \cos \lambda$$

$$Y = \sin \phi \cos \lambda \quad (A-1)$$

$$Z = \sin \lambda$$

b. Rectangular to spherical:

$$\lambda = \tan^{-1} \left(\frac{Z}{\sqrt{X^2 + Y^2}} \right) \quad (A-2)$$

$$\phi = \tan^{-1} (Y/X)$$

where

λ is latitude (positive north)

ϕ is longitude (positive west)

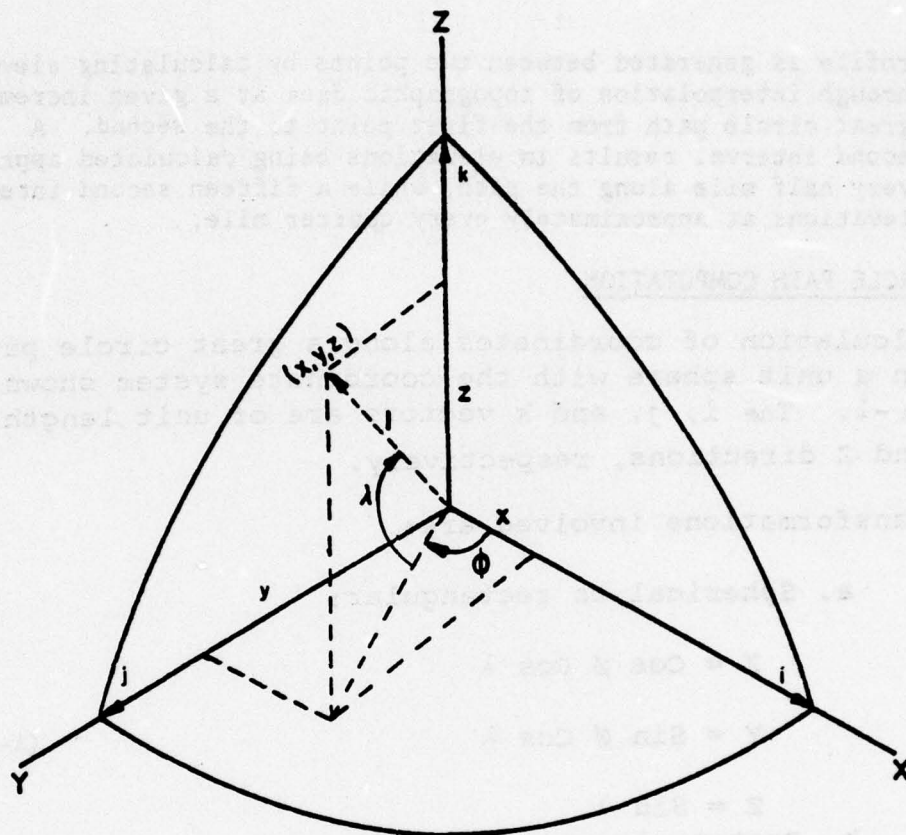


FIGURE A-1. COORDINATE SYSTEM FOR GREAT CIRCLE PATH CALCULATIONS.

Given a site shown in Figure A-2: as point S on the unit sphere (latitude λ_s , longitude ϕ_s), a vector \bar{S} from the center of the sphere to point S is constructed as:

$$\bar{S} = s_1 i + s_2 j + s_3 k \quad (A-3)$$

where s_1 , s_2 , and s_3 are scalars calculated from equation (A-1) with (λ_s, ϕ_s) . Similarly, a vector \bar{G} from the center of the sphere to a grid point $G(\lambda_G, \phi_G)$ is constructed:

$$\bar{G} = g_1 i + g_2 j + g_3 k \quad (A-4)$$

The vector \bar{P} from S to G is then

$$\begin{aligned} \bar{P} &= \bar{G} - \bar{S} = (g_1 - s_1) i + (g_2 - s_2) j + (g_3 - s_3) k \\ &= p_1 i + p_2 j + p_3 k \end{aligned} \quad (A-5)$$

The unit vector \bar{U} in the direction of \bar{P} is

$$\begin{aligned} \bar{U} &= \frac{\bar{P}}{|\bar{P}|} = \frac{p_1 i + p_2 j + p_3 k}{\sqrt{p_1^2 + p_2^2 + p_3^2}} \\ &= u_1 i + u_2 j + u_3 k. \end{aligned} \quad (A-6)$$

Equal spacing along the great circle path from S to G is achieved through the calculation of the angle γ_n (shown in Figure A-3) as follows:

$$\begin{aligned} \theta &= \cos^{-1} \left[\sin \lambda_s \sin \lambda_G + \cos \lambda_s \cos \lambda_G \cos (\phi_G - \phi_s) \right] \\ d_0 &= C \Delta_s \end{aligned} \quad (A-7)$$

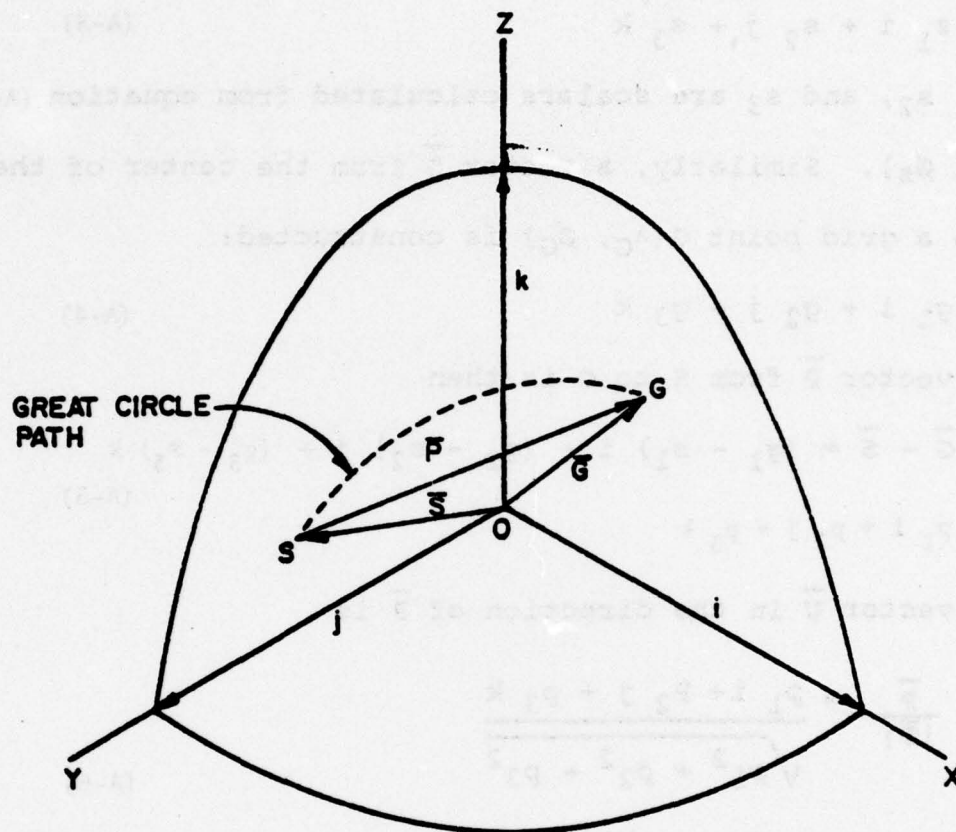


FIGURE A-2. THE VECTORS \vec{S} , \vec{G} , AND \vec{P} .

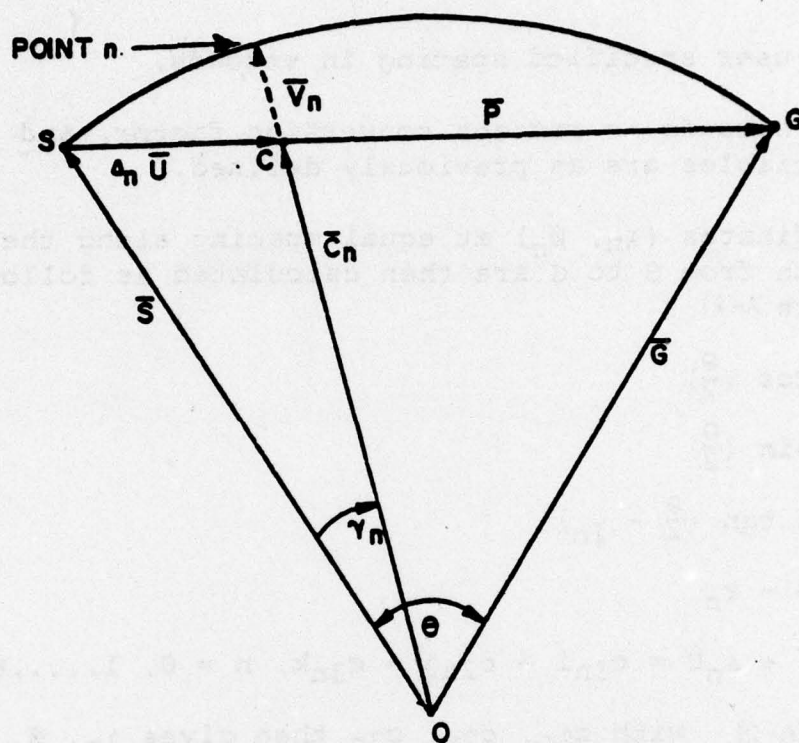


FIGURE A-3. SPACING ANGLE γ_n .

$$N = \left\langle \frac{\theta}{\Delta_0} + 1 \right\rangle$$

$$\gamma_n = \frac{\theta}{N} n, \quad n = 0, 1, \dots, N$$

where

Δ_s is user specified spacing in seconds,

C is seconds to radians conversion factor, and other variables are as previously defined.

Coordinates (λ_n, θ_n) at equal spacing along the great circle path from S to G are then calculated as follows (see Figure A-4) :

$$h = \cos \left(\frac{\theta}{2} \right) \quad (A-8)$$

$$b = \sin \left(\frac{\theta}{2} \right)$$

$$r_n = h \tan \left(\frac{\theta}{2} - \gamma_n \right)$$

$$\Delta_n = b - r_n$$

$$\vec{C}_n = \vec{S} + \Delta_n \vec{U} = c_{1n} \vec{i} + c_{2n} \vec{j} + c_{3n} \vec{k}, \quad n = 0, 1, \dots, N$$

Equation (A-2) with c_{1n} , c_{2n} , c_{3n} then gives λ_n , θ_n . These coordinates apply equally to the unit vector \vec{V}_n (Figure A-3), in the direction of \vec{C}_n , which is the vector from the center of the unit sphere to the n th point on the great circle path from S to G .

DATA RETRIEVAL

It is, of course, necessary to determine which topographic data block is needed to obtain the elevation of a given point, and then to retrieve it from drum. In order to minimize drum access time, two data blocks are always retained in computer memory during profile generation from a given site. One array is reserved for the block area containing the site location. The second array is used to store other data blocks which are required for grid points not contained in the site block area (See FIGURE A-5).

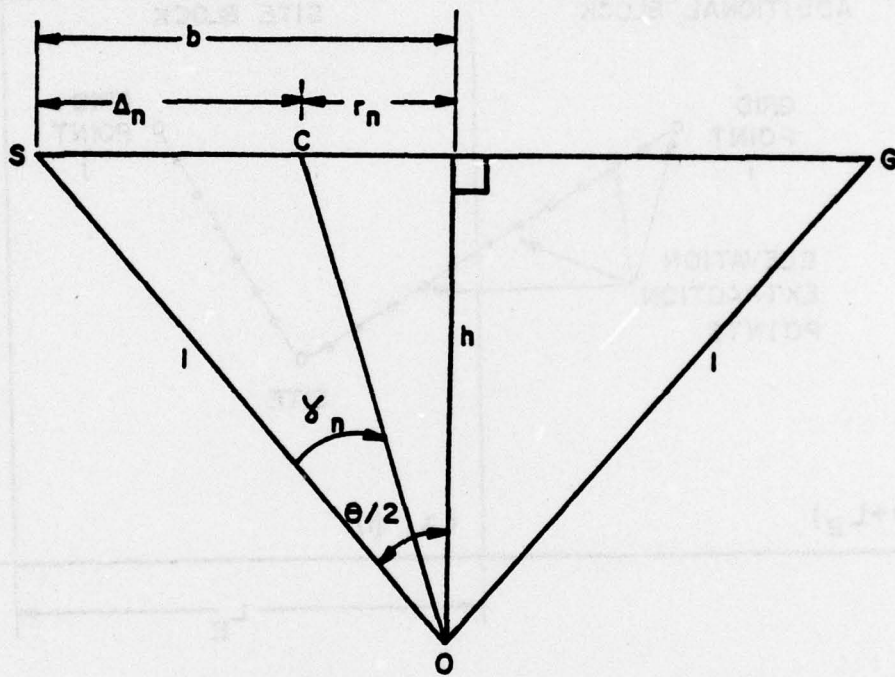


FIGURE A-4. THE SCALAR Δ_n .

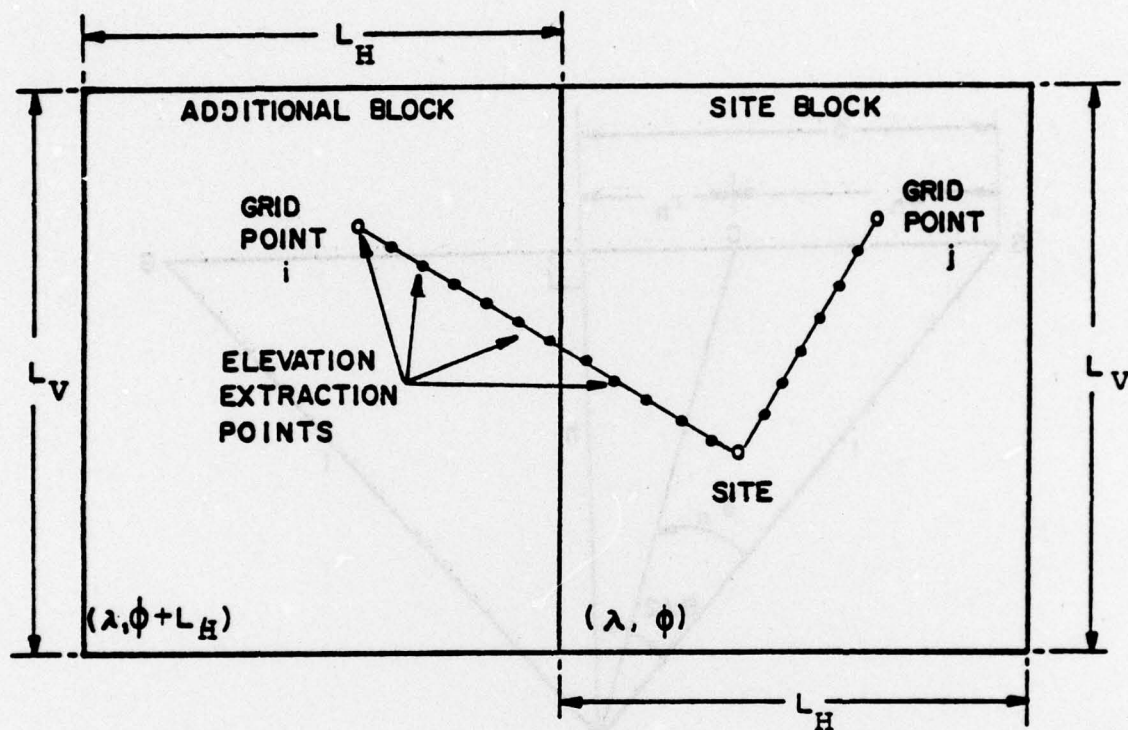


FIGURE A-5. DATA RETRIEVAL SITUATION.

Let λ_n, φ_n be the latitude and longitude (in radians) of the n^{th} point along the great circle path from the site to a given grid point. Since topographic data blocks are referenced by latitude and longitude of the southwest corner in minutes, the block reference $(\lambda_{sw}, \varphi_{sw})$ for λ_n, φ_n and additional measures (x, y) used in elevation extraction are determined as follows:

For $\lambda_n > 0$,

$$\lambda_{sw} = \lambda_T$$

$$y = \lambda_C - \lambda_T$$

For $\lambda_n \leq 0$,

$$\lambda_{sw} = \lambda_T - L_V$$

$$y = \lambda_C - \lambda_T + L_V$$

For $\varphi_n > 0$,

$$\varphi_{sw} = \varphi_T + L_H$$

$$x = \varphi_T - \varphi_C + L_H$$

For $\varphi_n \leq 0$,

$$\varphi_{sw} = \varphi_T$$

$$x = \varphi_T - \varphi_C$$

(A-9)

where

$\lambda_{sw}, \varphi_{sw}$ are latitude and longitude of the southwest corner in minutes,

λ_C, φ_C are latitude λ_n and longitude φ_n converted to minutes,

$$\lambda_T = \left\langle \frac{\lambda_C}{L_V} \right\rangle \cdot L_V \text{ in minutes,}$$

$$\varphi_T = \left\langle \frac{\varphi_C}{L_H} \right\rangle \cdot L_H \text{ in minutes,}$$

L_V, L_H are vertical and horizontal topographic block lengths in minutes,

x, y are extraction measures in minutes (see Figure A-6),

and $\langle \rangle$ denotes truncation to nearest integer value.

The necessary block reference point, $(\lambda_{sw}, \varphi_{sw})$, is then compared with those of the two data blocks in the computer. If the required data is already available, the elevation at λ_n , φ_n may then be extracted; if not present, the data is first retrieved from drum storage.

ELEVATION EXTRACTION

Two methods of deriving an elevation are available. The first procedure assumes the same elevation for the location in question as is recorded in the topographic data file for the location nearest to it. The second method performs a linear interpolation in two directions using the elevations of the four proximal points, with the resultant being the desired elevation.

The required row(s) and column(s) of the 121 x 121 topographic data matrix (see Reference 1) are first determined.

$$m = \langle M \rangle$$

$$n = \langle N \rangle$$

(A-10)

where

m is the number of the nearest matrix row south of the location in question,

n is the number of the nearest matrix column west of the location in question,

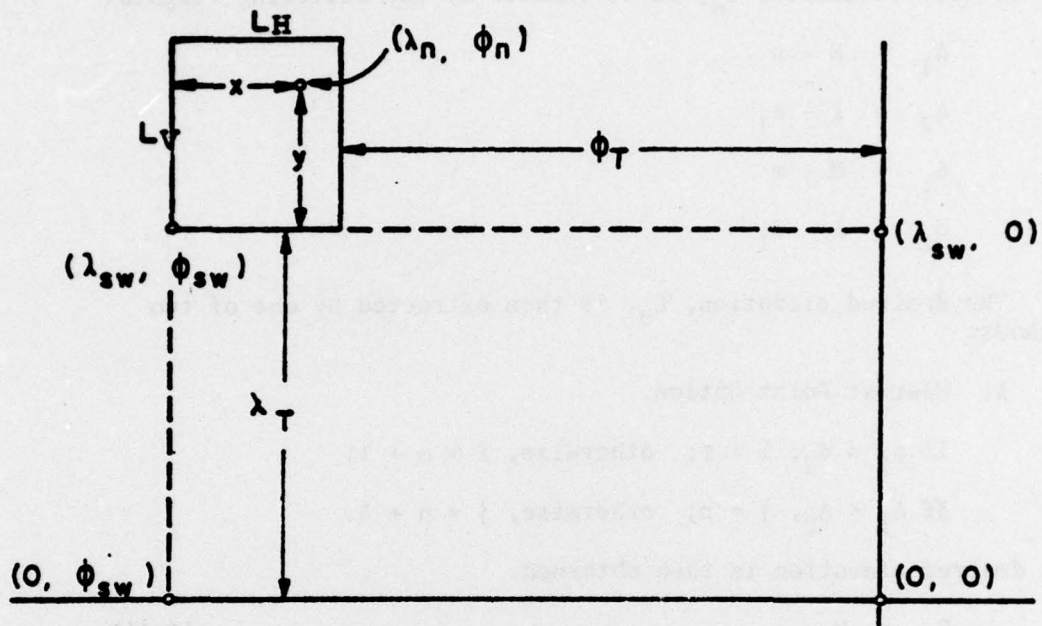


FIGURE A-6. SOUTHWEST CORNER DETERMINATION.

$$M = \frac{120 \cdot y}{L_V} + 1,$$

$$N = \frac{120 \cdot x}{L_H} + 1,$$

and other variables are as previously defined.

The relation of the proximal elevation shown in Figure A-7 to the desired elevation, E_d , is determined by the following weights:

$$\Delta_1 = N - n$$

$$\Delta_2 = 1 - \Delta_1$$

$$\delta_1 = M - m$$

$$\delta_2 = 1 - \delta_1$$

The desired elevation, E_d , is then extracted by one of two methods:

1. Nearest Point Option.

If $\delta_1 < \delta_2$, $i = m$; otherwise, $i = m + 1$;

If $\Delta_1 < \Delta_2$, $j = n$; otherwise, $j = n + 1$.

The desired elevation is then obtained.

$$E_d = H_{i,j} \quad (A-11)$$

2. Four Point Option.

Interpolation in the north-south direction gives

$$E_a = \delta_2 H_{m,n} + \delta_1 H_{m+1,n} \quad (A-12)$$

$$E_b = \delta_2 H_{m,n+1} + \delta_1 H_{m+1,n+1}$$

Interpolation in the east-west direction then results in the desired elevation,

$$E_d = \Delta_2 E_a + \Delta_1 E_b \quad (A-13)$$

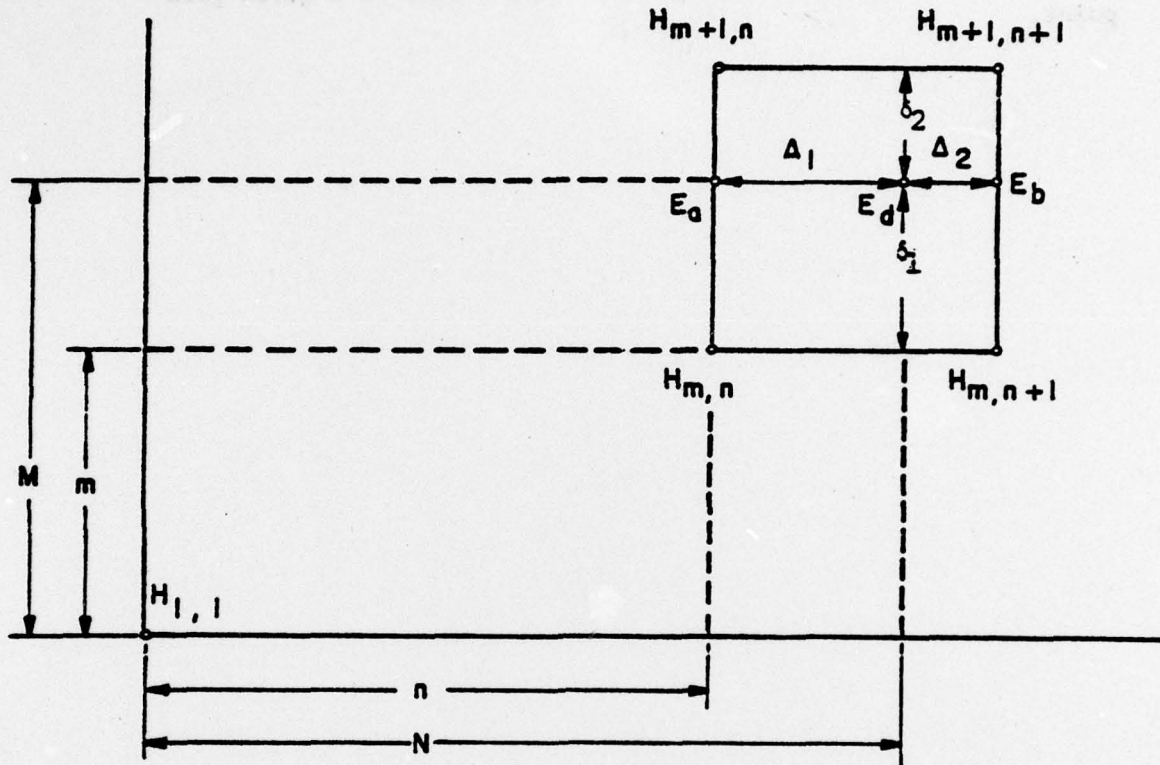


FIGURE A-7. ELEVATION DETERMINATION.

Since elevation values are initially biased by a quantity ΔE , quantized by a factor Q and stored in compressed form in the topographic data file, the elevation derived from either method must be converted to feet above sea level by the equation:

$$E = Q E_d + \Delta E \quad (A-14)$$

The extracted elevation, E , is thus one of any number of elevations that constitute an elevation profile from a site to a given grid point.

APPENDIX B

LINE-OF-SIGHT DETERMINATION

Before line-of-sight can be determined along a profile, two factors must be taken into consideration: the effect of refractivity, or bending, of radio waves as they are transmitted through the atmosphere, and the effect of earth's curvature with respect to the elevations.

Atmospheric refractivity is accounted for by assuming an effective earth's radius, $a_e = ka$ where k is the effective earth's radius factor and a is the true radius of the earth. This allows radio waves to be drawn as straight lines over an earth with effective radius a_e .

The value of k , for rays leaving a transmitter at low angles to the horizon, is given by the expression

$$k = \frac{1 + 10^{-6}N}{1 + 10^{-6}N + 10^{-6} a \Delta N} \quad (B-1)$$

where

N = local refractivity index

a = true earth's radius

ΔN = change in refractivity with respect to altitude.

In general, the change of refractivity varies exponentially and is given as $\Delta N = -7.32 e^{0.005577N}$. Taking $a = 6370$ kilometers yields

$$k = \frac{1 + 10^{-6}N}{1 + 10^{-6}N - 0.04663 e^{0.005577N}} \quad (B-2)$$

Assuming that the effective earth's radius is ka , each elevation, E_n , is now reduced by an appropriate amount, Δh_n , which corrects the elevation for earth's curvature. Using Figure B-1, it is shown that

$$\Delta h_n = -ka + \sqrt{(ka)^2 + X_n^2} \quad (B-3)$$

The arc length, dn , is a reasonable approximation for X_n as X_n differs from dn by less than one mile for path lengths up to 425 miles. A binomial series expansion of the radical in Equation B-3 yields the following equation

$$\Delta h_n = \frac{dn^2}{2ka} \quad (B-4)$$

The corrected elevation, E_c , is then

$$E_c = e_n - \frac{dn^2}{2ka} \quad (B-5)$$

The vertical angle, θ_n , is then calculated (Figure B-2) from the site to each corrected elevation along the profile by the following equation:

$$\theta_n = \tan^{-1} \left[\frac{E_c - (E_s + H)}{dn} \right] \quad (B-6)$$

where

E_s = site elevation above sea level (feet)

H = antenna height (feet).

As shown in Figure B-3 the limiting line-of-sight will occur at the values of dn and E_c which correspond to the maximum value of the vertical angle θ_n , or the radio horizon angle.

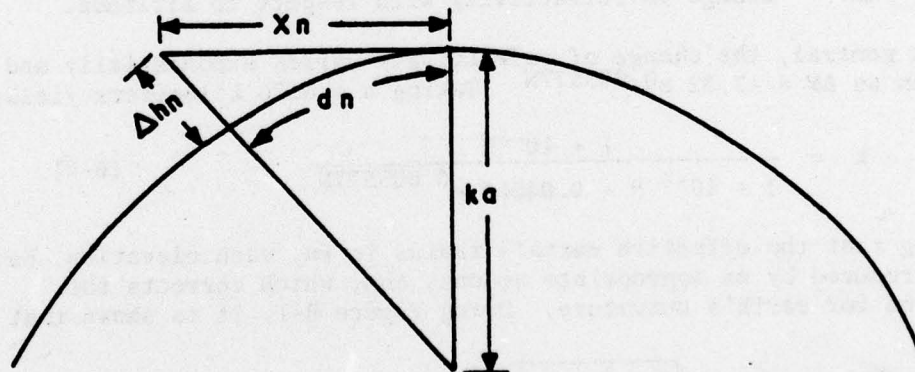


FIGURE B-1. ELEVATION CORRECTED FOR ATMOSPHERIC REFRACTIVITY AND EARTH'S CURVATURE.

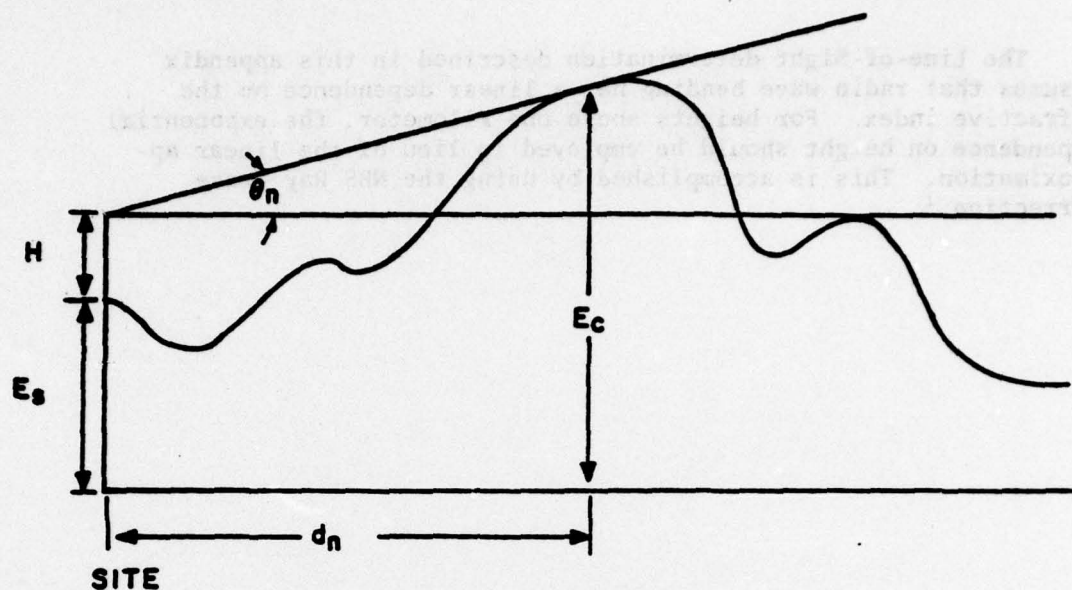


FIGURE B-2. RADIO HORIZON ANGLE, θ_n .

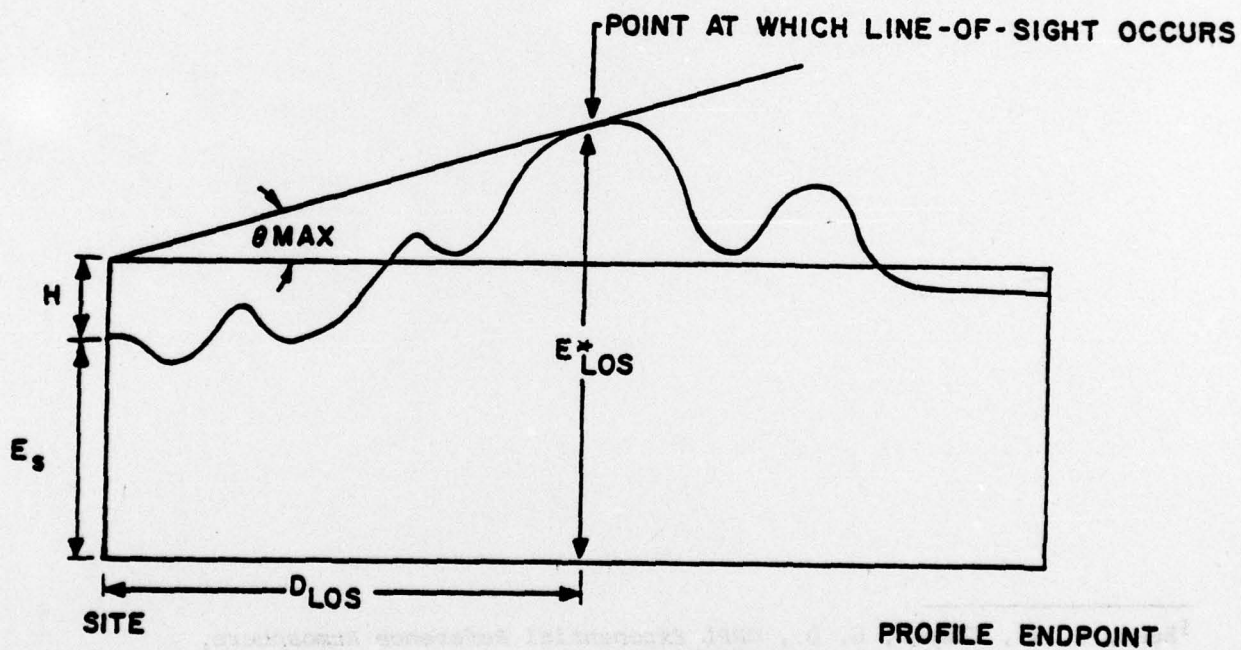
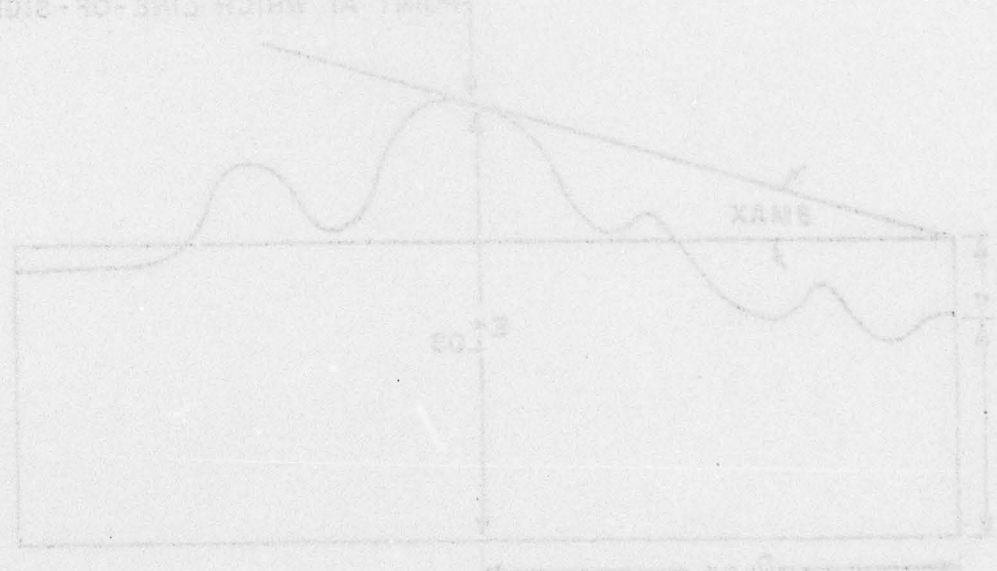
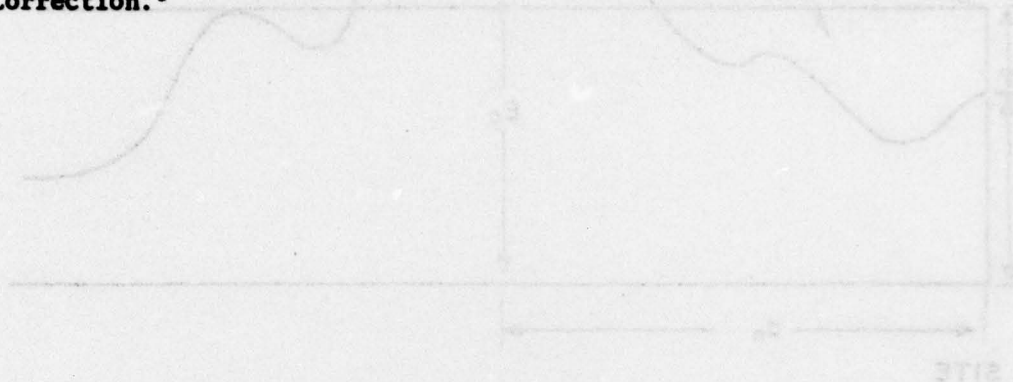


FIGURE B-3. DETERMINATION OF LINE-OF-SIGHT FOR A PROFILE.

The Line-of-Sight determination described in this appendix assumes that radio wave bending has a linear dependence on the refractive index. For heights above one kilometer, the exponential dependence on height should be employed in lieu of the linear approximation. This is accomplished by using the NBS Ray Trace Correction.¹



¹Bean, B. R., Thayer, G. D., *CRPL Exponential Reference Atmosphere*, NBS Monograph 4, October 29, 1959.